Non-linear Static Analysis of Masonry Structures by means of Equivalent Frames Simplified Approach

RIZZANO, G.¹; SABATINO, R.²

ABSTRACT:
Non-linear static analyses are the most effective way to predict force-displacement curves of masonry structures, due to the complex non linear behaviour of masonry panels which makes inadequate any kind of linear static analyses. Nevertheless, the choice of the appropriate model to use is a matter of paramount importance, as many aspects must be taken into account in order to reach a good approximation of the structural response. Many structural models aimed at predicting masonry buildings response have been proposed in recent years; these models often try to achieve a balance between the complexity of the model itself and the accuracy of obtained results.

Within this framework, the equivalent frame approach has been frequently adopted in order to simulate the response of masonry building.

This paper deals with the elaboration of the software FREMA (FRame Equivalent Masonry Analysis) aimed at predicting the force-displacement curve of masonry walls subject to in-plane forces. A procedure for the non-linear static analysis under displacement control has been developed; geometrical and mechanical non-linearity have been taken into account. The computer program herein developed has been validated by means of a comparison with experimental and numerical data available in literature. A further comparison between the proposed model and accurate models (FEM models) available in literature has been developed, in order to investigate the accuracy of the model and to underline the main approximations introduced by the simplified approach.

Keywords: Non linear static analysis, unreinforced masonry brickwork, FEM, equivalent frame.

1 INTRODUCTION

Traditional and historical masonry buildings typically constitute the majority of the building stock in almost all countries. In Italy, the last L'Aquila earthquake (April 2009) clearly showed that such buildings, often designed to resist under gravity loads only, experience severe damages under seismic events.

For these reasons, the assessment of the existing buildings and hence the evaluation of the vulnerability (i.e. the extension of the damage for different earthquake scenarios) of the building stock is one of the most important goals of modern structural engineering and it is a fundamental tool to investigate the priorities of the retrofit work.

However, the evaluation of the vulnerability requires adequate tools in order to be performed properly. From this point of view, the adoption, in last years, of “performance-based” earthquake engineering concepts led to a massive use of non-linear static procedures in order to evaluate the seismic behaviour of masonry structures. Aiming at this, different strategies of modelling may be adopted to evaluate the response of structures in terms of strength and ultimate displacement. Among

¹) Associate Professor, University of Salerno, Department of Civil Engineering, g.rizzano@unisa.it
²) Ph.D. Student, University of Salerno, Department of Civil Engineering, rsbatino@unisa.it
them, two main approaches can be identified: the Finite Element Method (FEM) approach and the equivalent frame (EqF) approach.

In the first approach, the masonry continuum is discretized into a certain number of finite elements; accurate constitutive laws can be adopted and the model can provide an accurate prediction of the behaviour of the structure: the FEM method can actually be a very powerful analysis tool. Nevertheless, the FEM method suffers from high computational effort, it can be time-consuming, exhibits a strong mesh-dependency and sometimes requires an accurate calibration of the input parameters (calibration "ex-post").

The second approach is based on the adoption of "equivalent frames", a model very common to structural engineers. The structure is described by an assemblage of vertical and horizontal elements (piers and spandrels) and rigid offsets; each element is modelled by proper force-displacements laws in order to take into account mechanical non-linearity. This approach introduces strong simplifications, and hence its accuracy clearly depends on the consistency between the adopted hypotheses and the actual structural problem.

From these preliminary remarks, it is clear that, in order to deal with a large stock of buildings, FEM models adoption can be burdensome, and, above all, it requires high level practitioners who completely master the method. The equivalent frame model, on the other hand, can be a simple and effective tool, especially for practical purposes, but its accuracy and its most critical issues must be carefully investigated.

This paper makes a contribution to the seismic analysis of masonry buildings by proposing a simplified analytical model able to develop, within the framework of equivalent frame models, the non linear static analyses of masonry structures.

2 A MODEL FOR THE NON-LINEAR STATIC ANALYSIS OF MASONRY WALLS

2.1. Description and hypotheses of the proposed model

The model herein described is able to perform the non-linear static analysis of 2D masonry walls subject to in-plane forces under displacement control. The main features and assumptions of the proposed model are herein summarized.

Displacement control approach: the analysis is performed under displacement control. This is a fundamental feature of this model, as the seismic assessment of masonry buildings can be performed by a number of non-linear static procedures (the coefficient method [1], the capacity spectrum method [2, 3], the N2 method [4]) which generally require to compare the seismic demand and the building capacity in terms of displacements. In particular, this comparison is achieved by substituting the actual building response by an equivalent single-degree-of-freedom (SDOF) oscillator [5]. From these remarks, it is clear that the prediction of the complete capacity curve is of paramount importance in seismic analyses, and this can be achieved only with an approach conducted under displacement control.

Geometry of the wall: the structural model represents, by means of an equivalent frame, every kind of plane masonry wall, with different openings distribution. The wall is idealized by an assemblage of vertical elements (piers), horizontal elements (spandrels) and rigid offsets. In addition, reinforced concrete ring beams can be added as horizontal elements. The length of the rigid offsets is a very important matter in modelling equivalent frames, because of the stiffness given to the structure. In this paper, with reference to the validation of the proposed model, the empirical approach proposed by Dolce [6] is taken into account.

Non linearity approach: in literature, within the equivalent frame approach, mechanical non linearity has been traditionally modelled considering flexural springs located at the ends of the piers and of the spandrels and translational shear springs located at the mid-span of piers and spandrels. In this work, in order to obtain more accurate results, a spread plasticity approach is considered, i.e. each element
(spandrel or pier) is discretized into a number of slices. Each slice is then modelled in terms of moment-curvature and shear-shear strain curves; the overall behaviour of each element is thus obtained by combining the contributions of each slice constituting the element.

On top of that the proposed model updates the nodal coordinates during the analysis, so that geometrical non linearity is taken into account.

**Global equilibrium**: the proposed model is formulated by respecting local equilibrium of each element of the frame and global equilibrium of the frame itself at each step of the analysis. Thus the proposed model provides a more accurate equilibrium approach in comparison with the classic POR-like methods [7], that generally assume a storeys-equilibrium approach.

**Collapse of the structure**: the proposed model is able to predict the force-displacement curve of masonry walls by considering the progressive damage and failure of lateral load resisting elements, as will be described in the following. For this reason, if a softening branch of the overall force-displacement curve is present, according to Eurocode 8 [8], the collapse condition is attained when the base shear has dropped below the 80% of the peak resistance.

### 2.2. Constitutive laws of the elements

Although the elements undergo flexural and shear deformation, the proposed model does not consider a combined triaxial interaction rule between axial force \( N \), bending moment \( M \) and shear \( V \). The model only considers two separate biaxial interaction between axial forces and bending moments (\( N-M \) domain) and axial forces and shear (\( N-V \) domain).

**\((N-M)\) and \((N-V)\) domains of piers**: The strength in terms of ultimate moment has been defined by considering the uniaxial compressive stress-strain behaviour, modelled by the general equation [9]:

\[
\frac{\sigma}{\sigma_d} = A \left( \frac{\varepsilon}{\varepsilon_d} \right) + B \left( \frac{\varepsilon}{\varepsilon_d} \right)^C
\]  

(1)

being \( \sigma_d \) the design uniaxial compressive strength, \( \varepsilon_d \) the strain corresponding to the attainment of \( \sigma_d \). The above relation is able to express most of the proposed stress-strain laws found in literature by a proper choice of the values of \( A \), \( B \) and \( C \) (Turnsek-Sheppard [10], Hendry [11]). It is not useless to point out that the proposed law (Figure 1) can express a softening branch, if the curve is plotted up to the ultimate strain \( \varepsilon_u > \varepsilon_d \). In this paper, according to Tassios proposal [12], the following values have been adopted: \( A = 2, B = -1, C = 2 \).

![Stress-strain law. Uncracked and cracked masonry panel cross-section](image)

**Figure 1.** Stress-strain law. Uncracked and cracked masonry panel cross-section
By considering the rotational and translational equilibrium equations of a masonry panel cross-section (Figure 1), the complete moment-curvature ($M-\chi$) relationship, for a given value of the axial force $N$, can be obtained both for uncracked and cracked cross-section [9]:

**Uncracked Section**

$$\nu = k_1 (2\bar{\xi} - 1) + k_2 \left[ \bar{\xi}^{C+1} - (\bar{\xi} - 1)^{C+1} \right]$$

$$\mu = \frac{1}{2} \left( \frac{1}{2} - \bar{\xi} \right) (2\bar{\xi} - 1) + \frac{2}{3} k_1 \left[ \bar{\xi}^3 - (\bar{\xi} - 1)^3 \right] +$$

$$+ k_2 \left( \frac{1}{2} - \bar{\xi} \right) \left[ \bar{\xi}^{C+1} - (\bar{\xi} - 1)^{C+1} \right] + k_2 \left[ \frac{C+1}{C+2} \bar{\xi}^{C+2} - (\bar{\xi} - 1)^{C+2} \right]$$

**Cracked Section**

$$\nu = k_1 \bar{\xi}^2 + k_2 \bar{\xi}^{C+1}$$

$$\mu = k_1 \left( \frac{1}{2} - \bar{\xi} \right) \bar{\xi}^2 + \frac{2}{3} k_1 \bar{\xi}^3 + k_2 \left( \frac{1}{2} - \bar{\xi} \right) \bar{\xi}^{C+1} + k_2 \frac{C+1}{C+2} \bar{\xi}^{C+2}$$

being $\bar{\xi} = x/D$ the normalised neutral axis, $\nu = N/Dt\sigma_d$ the normalised axial stress, $\mu = M/tD^2\sigma_d$ the normalised bending moment, $k_1$ and $k_2$ two coefficients depending on the on the curvature $\chi$:

$$k_1 = \frac{A}{2} \left( \frac{xD}{\varepsilon_d} \right)$$

$$k_2 = \frac{B}{C+1} \left( \frac{xD}{\varepsilon_d} \right)^C$$

The pier flexural strength depends on the axial force acting upon the pier; the flexural collapse corresponds to the attainment of the ultimate strain $\varepsilon_u$ on the extreme fibre of the cross-section.

It is useful to point out that, if during the analysis a pier is subjected to tension, the pier is removed from the structural scheme, its weight is assigned to the corresponding inferior pier and the equivalent frame scheme is globally updated.

The (N-V) domain of piers is considered as a bilinear elastic-perfectly plastic curve (Figure 2). The strength in terms of shear is expressed by comparing the minimum between shear failure with diagonal cracks $V_u^d$ and shear failure with sliding $V_u^s$. For shear failure with sliding a Coulomb approach has been considered:

$$V_u^s = (c + \mu p) D't.$$

For shear failure with diagonal cracks the model proposed by Mann-Müller [13] and Magenes-Calvi [14] has been adopted for masonry made up by a regular pattern of bricks and mortar joints:
$V_u^d = Dt\tau_u$ \hspace{1cm} (8) \\
with \tau_u = \min(\tau_{uw}, \tau_b) \\
\tau_{uw} = \frac{c + \mu p}{1 + \alpha_V} \\
\tau_b = \frac{f_{bt}}{2.3(1 + \alpha_V)} \sqrt{1 + \frac{p}{f_{bt}}}$

Finally, the model proposed by Turnsek-Cacovic [15] and later modified by Turnsek-Sheppard [10] has been considered for irregular and historical masonry walls, where a regular brick pattern cannot be found:

$V_u^d = \frac{1.5f_{vd,0}Dt}{b} \sqrt{1 + \frac{p}{1.5f_{vd,0}}} \hspace{1cm} (9)$

being $c$ the cohesion, $\mu$ the friction coefficient, $p$ the medium compressive stress acting on the cross section, $D$ and $D'$ respectively the pier width and the reacting width, $t$ the pier thickness, $\alpha_V$ the shear coefficient, $f_{bt}$ the brick design tension strength, $f_{vd,0}$ the design shear strength with no axial force, $b$ a coefficient related to the pier geometrical ratio. The shear collapse corresponds to the attainment of the maximum displacement $\delta_u$, assumed equal to 0.4% of the deformable height of the pier (according to the Italian Code [16]).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{constitutive_laws.png}
\caption{Constitutive laws of piers and spandrels}
\end{figure}

**N-M and N-V domains of spandrels**: In the proposed model the flexural behaviour of spandrels, for this preliminary phase of the FREMA code development, is modelled by means of an elastic-perfectly plastic relation (Figure 2). The ultimate moment is expressed by the equation:

$M_u = \frac{f_{vd,0}t_s h_s^2}{6} \hspace{1cm} (10)$

where $f_{vd,0}$ is the design shear strength with no axial force, $t_s$ is the spandrel thickness, $h_s$ the spandrel height.
The shear behaviour of the spandrel is modelled as elastic with a brittle failure at the attainment of $V_u$ with a residual strength after cracking [17] equal to 0.25 $V_u$ (Figure 2). The expression of $V_u$ is given by the equation:

$$V_u = f_{\text{id}} g f_s h_s$$  \hspace{1cm} (11)

### 2.3. Algorithm description

As already stated, the structural masonry walls, made up by piers and spandrels, are modelled as an equivalent frame. It is useful to stress that the proposed model deals with 2D walls, so that only in-plane rotations and displacements are taken into account. In the following a brief description of the proposed algorithm to achieve a displacement control procedure is supplied.

In order to take into account geometrical and mechanical non-linearity, the procedure must be performed considering a structure secant stiffness matrix, i.e. by taking into account the actual stress acting on the structural elements, according to the constitutive laws previously described.

Preliminarily, the problem is generally approached by considering the well known equation:

$$\{F\}_i = [K_{\text{sec}}]^{-1} \{\delta\}_i$$  \hspace{1cm} (12)

being $\{F\}_i$ and $\{\delta\}$ respectively the horizontal forces vector and the displacement vector, $[K_{\text{sec}}]^{-1}$ the frame secant stiffness matrix. It is worth to underline that the index $i$ and $i-1$ refer to the convergence procedure described in the following. The secant stiffness matrix is obtained by considering the assemblage of piers and spandrels. Each pier and spandrel is, in turn, discretized by a number $N_{\text{slices}}$ of slices (according to a smeared plasticity approach); the stiffness of each elements is thus obtained by computing the deformability coefficients of each slice, and by properly combining them, according to procedure described in [18,19].

It is useful to remark that for the first step of the analysis, the secant stiffness matrix coincides with the elastic stiffness matrix. In the following steps, the secant stiffness matrix is obtaining by updating the flexural and shear stiffness of the elements as a function of the current stress.

In particular, dividing the vector of force $\{F\}_i$ into the fixed part $\{F\}_f$ and the variable part $\alpha_i \{S\}$ (basically, the distribution of lateral forces):

$$\{F\}_i = \alpha_i \{S\} + \{F\}_f = [K_{\text{sec}}]^{-1} \{\delta\}_i$$  \hspace{1cm} (13)

The proposed procedure develops by setting the displacement $\delta_c$ of the control node and finding the coefficient $\alpha$ corresponding to it. The procedure is iterative, as already stated.

1. For $\alpha=0$, the displacement vector $\{\delta_0\}$, corresponding to the fixed loads $\{F\}_f$ is obtained:

$$\{\delta_0\}_i = [K_{\text{sec}}]^{-1} \{F\}_f$$  \hspace{1cm} (14)

2. By using eq. (12)-(14) the vector $\{\delta - \delta_0\}_i$, i.e. the displacement vector due to the variable part of the force vector, can be obtained:
\[ \{\delta - \delta_0\}_j = \alpha_i [K_{sec}]^{-1}_{j-1} \{S\} \] (15)

3. If a different value of \( \alpha = \alpha_{1,i} \) (for instance one could set \( \alpha_{1,i} = 1 \)) is hence considered, the corresponding displacement vector \( \{\delta - \delta_0\}_i \) is obtained as:

\[ \{\delta - \delta_0\}_i = \alpha_{1,i} [K_{sec}]^{-1}_{i-1} \{S\} \] (16)

4. By using equations (12), (15) and (16), and extracting the displacements \( \delta_{0,i} \) and \( \delta_{1,i} \) of the control node, the coefficient \( \alpha_i \) corresponding to \( \delta_c \) can be eventually evaluated:

\[ \delta_c = \delta_{0,i} + \frac{\alpha_i}{\alpha_{1,i}} \left( \delta_{1,i} - \delta_{0,i} \right) \Rightarrow \alpha_i = \frac{\delta_c - \delta_{0,i}}{\delta_{1,i} - \delta_{0,i}} \] (17)

so that the displacement vector \( \{\delta\}_i \) is known, and the problem expressed from eq. (13) is finally solved. In particular, the updated secant stiffness matrix \([K_{sec}]_i\) is known. The found \( \alpha_i \) is the actual value of \( \alpha \) corresponding to \( \delta_c \) if:

\[ \|[K_{sec}] - [K_{sec}]_{i-1}\| < \varepsilon \] (18)

being \( \varepsilon \) the desired tolerance, otherwise an iteration on the index \( i \) must be performed, repeating the procedure described by equations (15)-(18) and by assuming, at the beginning of the new iteration, \([K_{sec}]_i = [K_{sec}]_i\).

3 PRELIMINARY VALIDATION OF THE MODEL

The model herein described has been validated by comparing the model prediction with a very detailed experimental test carried out at the University of Pavia, where a two storey full-scale masonry building prototype has been experimented by Calvi and Magenes [20]. The building, having a 6x4.4 m plan and 6.4 m height, contains an independent wall (named “door-wall”) in-plane loaded, which thickness is 25 cm. The experimental test has been carried out by imposing a constant vertical load on each storey (248.8 kN at the first floor, 236.8 kN at the second floor), while the seismic forces were simulated by applying two concentrated forces at the floor levels, by imposing a displacement cyclic history simulating a uniform load pattern.

Figure 3. Pavia door-wall. Geometry and equivalent frame model
The equivalent frame model has been defined, on the basis of the wall geometry depicted in Figure 3, considering 9 nodes (3 clamped and 6 representing the rigid panel zone), 6 piers and 4 spandrels. Piers and spandrels have been discretized into slices and have been modelled by means of the constitutive laws described in section 2.2. of this paper.

The material properties adopted for the equivalent frame model are the following: masonry Young’s modulus $E=1400$ MPa, masonry shear modulus $G=480$ MPa, compressive strength of masonry $\sigma_d=6.2$ MPa, cohesion $c=0.23$ MPa, friction coefficient $\mu=0.58$, block tensile strength $f_{bt}=1.22$ MPa, block aspect ratio equal to $\frac{1}{4}$.

In Figure 4a, the comparison among the experimental result, the proposed model, the equivalent frame model proposed by the University of Pavia - SAM [17] - and an accurate FEM simulation proposed by the same research group is depicted. It is very important to underline that the experimental curve has been plotted as the monotonic envelope curve of the structural response.

The comparison clearly shows a satisfactory agreement among the proposed model and the experimental response. Furthermore, with reference to both the SAM code and the FEM simulation, a good agreement is present. In particular it is useful to point out that prediction of the ultimate strength is the same for the models and only a slight overestimation of the global response is provided by the proposed model. This can be explained considering that the model prediction is related to a monotonic loading process, while the experimental test have been carried out by means of a cyclic loading history, and hence it represents the lower bound of the monotonic response.

![Figure 4. Overall total base shear-top displacement curves a) Door Wall, b) Via Martoglio Wall](image)

A further validation of the model has been performed by considering the results of a wide Italian nationwide research project, the so-called “Catania Project” [21]. The “Catania Project” was focused on the analysis of the seismic performance of two existing masonry buildings, which were analysed by several Research Groups, each of one adopting different advanced software packages (the Pavia R. G. used the SAM code, the Genoa R.G. used an accurate FEM model [22] while the Basilicata R.G. used a no tensile strength macroelement model with shear failure and crushing control [23]).

The first analyzed wall (“Via Martoglio” wall) has been extracted from a five-storeys existing buildings; the wall has been analyzed by means of the FREMA code adopting the following material properties: masonry Young’s modulus $E=1600$ MPa, masonry shear modulus $G=300$ MPa, compressive strength of masonry $\sigma_d=6.0$ MPa, cohesion $c=0.15$ MPa, friction coefficient $\mu=0.50$, block tensile strength $f_{bt}=1.0$ MPa.

In Figure 4b the comparison between the FREMA model and the several Italian R.G. model predictions is depicted. The comparison shows a good agreement between the proposed model and the SAM code; furthermore, a good matching in terms of residual strength can be found with the Genoa R.G. model.
A further comparison has been made for the two walls extracted from the “Via Verdi” building [21]. The mechanical parameters adopted for the analyses were: masonry Young’s modulus $E=1500$ MPa, masonry shear modulus $G=250$ MPa, compressive strength of masonry $\sigma_d=2.4$ MPa, cohesion $c=0.20$ MPa, friction coefficient $\mu=0.50$.

For these walls, in addition to the model predictions of the Genoa, Pavia and Basilicata R.G., further equivalent frame models developed by Pasticier et al. [24] by using the SAP2000 V.10 code are available. The comparison among the FREMA code and the other simulations are depicted in Figure 5.

The result of the FREMA prediction is very similar to that provided by the SAM code and the Pasticier simulations for both the analyzed walls. In particular, with reference to the Wall D, the proposed model seems to be in accordance with the above models both in terms of ultimate shear strength and in terms of ultimate displacement prediction.

Figure 5. Overall total base shear-top displacement curves (Via Verdi walls): a) Wall A, b) Wall D.

4 CONCLUSIONS

In this paper the software FREMA aimed at the analysis of masonry walls in-plane loaded by means of the equivalent frame approach [25] has been presented. The main assumptions of the model regarding the spread plasticity approach, the constitutive laws of piers and spandrels, the global equilibrium and the conventional collapse conditions and a brief description of the solving algorithm have been discussed. The proposed model has been preliminarily validated by means of a comparison with experimental results and FEM accurate simulations available in literature, showing a satisfactory agreement.

REFERENCES


